## C3 Jan07 - Solutions

1 a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta(1-\sin^2\theta) - \sin^3\theta$

$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ #

b) $\sin\theta = \frac{\sqrt{3}}{4} \Rightarrow \sin 3\theta = \frac{3\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3$

$= \frac{3\sqrt{3}}{4} - 4\times\frac{3\sqrt{3}}{64}$

$= \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$

2 a) $f(x) = 1 - \frac{3}{(x+2)} + \frac{3}{(x+2)^2}$

$= \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2}$

$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2}$



$= \frac{x^2+x+1}{(x+2)^2}$ #

b) $x^2+x+1 = (x+\frac{1}{2})^2 - \frac{1}{4} + 1$

$= (x+\frac{1}{2})^2 + \frac{3}{4} > 0$ #

c) Numerator $>0$ from b)

Denominator $>0 \Rightarrow f(x) > 0$. #

3 a) $x = 2\sin y$

when $y = \pi/4$ $x = 2\sin\pi/4 = 2\times\frac{\sqrt{2}}{2} = \sqrt{2}$ #

b) $\frac{dx}{dy} = 2\cos y \Rightarrow \frac{dy}{dx} = \frac{1}{2\cos y}$

When $y = \pi/4$ $\frac{dy}{dx} = \frac{1}{\cos\pi/4} = \frac{1}{2\times\sqrt{2}/2} = \frac{1}{\sqrt{2}}$ #

c) Grad of tangent at P $= \frac{1}{\sqrt{2}}$

$\therefore$ Grad of normal $= -\sqrt{2}$

$\therefore$ eqn. of normal $y - \pi/4 = -\sqrt{2}(x - \sqrt{2})$



$y - \pi/4 = -\sqrt{2}x + 2$

$y = -\sqrt{2}x + 2 + \pi/4$

4. I) $y = x(9+x^2)^{-1}$

turning points when $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = (9+x^2)^{-1} + x\times -(9+x^2)^{-2}\times 2x$

$= (9+x^2)^{-1} - 2x^2(9+x^2)^{-2}$

$\therefore \frac{1}{9+x^2} - \frac{2x^2}{(9+x^2)^2} = 0$

$\frac{9+x^2-2x^2}{(9+x^2)^2} = 0$

$9 - x^2 = 0$

$x = \pm 3$

When $x = 3$ $y = \frac{3}{9+(3)^2} = \frac{3}{18} = \frac{1}{6}$

$\therefore$ Coords $(3, \frac{1}{6})$ and $(-3, -\frac{1}{6})$



II) $y = (1+e^{2x})^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{1/2}\times 2e^{2x}$

$\frac{dy}{dx} = 3e^{2x}(1+e^{2x})^{1/2}$

$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}\ln 3} = 3e^{\ln 3}(1+e^{\ln 3})^{1/2} = 9(1+3)^{1/2}$

$= 18$

5 a) $y = \sqrt{3}\cos x + \sin x$

$R\sin(x+\alpha) = R\sin x\cos\alpha + R\sin\alpha\cos x$

$\therefore R\cos\alpha = 1$ $R\sin\alpha = \sqrt{3}$

$\tan\alpha = \sqrt{3}$

$\alpha = \pi/3$

$\therefore R = \frac{1}{\cos\pi/3} = 2$

$\therefore y = 2\sin(x + \pi/3)$

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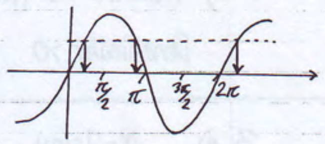
b) $1 = 2\sin\left(x+\frac{\pi}{3}\right) \quad 0 < x \le 2\pi$

$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$

$x+\frac{\pi}{3} = \sin^{-1}\frac{1}{2}$

$x+\frac{\pi}{3} = \frac{\pi}{6} \qquad \frac{\pi}{3} \le x+\frac{\pi}{3} \le \frac{7\pi}{3}$




$x+\frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$

$x = -\frac{\pi}{6}, \frac{3\pi}{6}, \frac{11\pi}{6}$

$\underline{x = \frac{\pi}{2}, \frac{11\pi}{6}}$

6. a) $y = \ln(4-2x)$

$e^y = 4-2x$

$2x = 4-e^y$

$x = 2-\frac{1}{2}e^y \quad \therefore \underline{f^{-1}: x \mapsto 2-\frac{1}{2}e^x} \quad \underline{x \in \mathbb{R}}$

b) $x < 2$.

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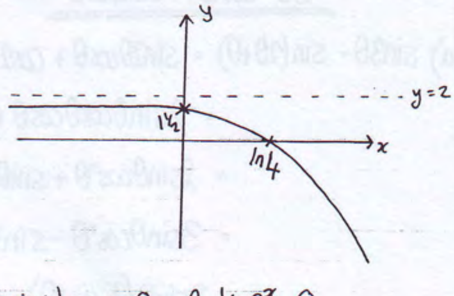
c)




meet $x$ axis when $y=0 \quad 2-\frac{1}{2}e^x = 0$

$e^x = 4$

$\underline{x = \ln 4}$

d) $x_1 = -\frac{1}{2}e^{-0.3} = \underline{-0.3704 \text{ (4dp)}}$

$x_2 = -\frac{1}{2}e^{-0.370409} = \underline{-0.3452 \text{ (4dp)}}$

e) $x_3 = -0.3540$
$x_4 = -0.3509$
$x_5 = -0.3520$
$x_6 = -0.3516$
$x_7 = -0.3517 \quad \therefore \quad \underline{x = -0.352 \text{ (3dp)}}$.

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7. a) $f(-2) = (-2)^4 - 4\times -2 - 8 = 16$

$f(-1) = (-1)^4 - 4\times 1 - 8 = -3$

$\therefore$ root exists between $-1$ and $-2$. #

b) turning point when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 4x^3 - 4 = 0$

$x^3 - 1 = 0$

$x^3 = 1 \Rightarrow x = 1 \Rightarrow y = 1^4 - 4\times 1 - 8 = -11$

$\underline{\text{Coords } (1, -11)}$

c)

$$\begin{array}{r} x^3+2x^2+4x+4 \\ x-2 \overline{\big)\, x^4 \qquad\quad -4x-8} \\ \underline{x^4-2x^3 \qquad\qquad\quad} \\ 2x^3 \quad -4x-8 \\ \underline{2x^3-4x^2 \qquad\quad} \\ 4x^2-4x-8 \\ \underline{4x^2-8x \quad} \\ 4x-8 \\ \underline{4x-8} \end{array}$$

$\therefore \underline{a=2, b=4, c=4}$

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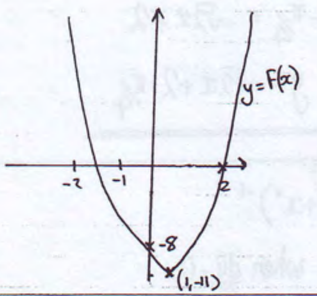
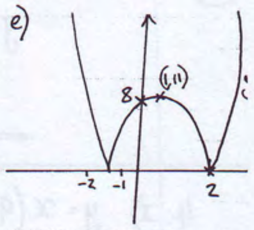
d)




e)




8 I) $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$

LHS $= \sec^2 x - \operatorname{cosec}^2 x$

$= \tan^2 x + 1 - (\cot^2 x + 1)$

$= \tan^2 x + 1 - \cot^2 x - 1 = \tan^2 x - \cot^2 x$ #

II) $y = \arccos x$

a) $x = \cos y$

$x = \sin\left(\frac{\pi}{2} - y\right)$

$\underline{\frac{\pi}{2} - y = \arcsin x}$

b). $\arccos x + \arcsin x$

$= y + \frac{\pi}{2} - y$

$= \underline{\frac{\pi}{2}}$